J_x is concentrated at the center of the conductor so that its separation from the boundary is $t/2$. The integral converges as a result and assumes the value

$$S_{xx,1}^{\phi}(x_i,x_i)=\frac{k_t}{4\omega\epsilon}H_1^{(2)}\left(k_t\frac{\Delta}{2}\right)\cdot\frac{2}{\Delta}I_{xi} \tag{B8}$$

which is independent of t provided that $t \ll \Delta$.

Finally, when a current density $J_\beta(x)$ is unbounded on a segment, as is the case for $J_z(x)$ at the edges of the conducting strip, the validity of its approximation by a constant value should be examined. The two integrals which concern us then are $S_{zz,1}^{c}$ and $S_{xz,1}^{\phi}$ evaluated on an edge segment. Under the approximation of the constant current density, the first integral has the finite value given in (B6) while the second integral vanishes as in (B4). We may assume that the error introduced in the first integral is smaller than its true value and can thus be neglected. The error introduced in the second integral, however, is equal to its true value and may not, therefore, be ignored. To evaluate $S_{xz,1}^{\phi}$ on an edge segment, we assume a current density J_z that satisfies the edge condition. We may thus write

$$J_z(x)=\frac{1}{2}\left[\Delta\left(\frac{\Delta}{2}-x\right)\right]^{-1/2}I_{zN}. \tag{B9}$$

Using (B9) in (B1) and applying the small argument approximation of the function $H_1^{(2)}$, we arrive, after the evaluation of some standard integrals, at the result

$$S_{xz,1}^{\phi}(x_N,x_N)=-j\frac{k_zk_t}{4\omega\epsilon}\frac{1}{2}FI_{zN} \tag{B10}$$

where

$$F=j\sqrt{2}\,\frac{0.6366}{k_t\Delta}\ln(3-2\sqrt{2}). \tag{B11}$$

References

[1] E. J. Denlinger, "A frequency dependent solution for microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 30–39, Jan. 1971.

[2] P. Silvester, "TEM wave properties of microstrip transmission lines," *Proc. Inst. Elec. Eng.*, vol. 115, pp. 43–48, Jan. 1968.

[3] R. E. Collin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1960, pp. 23.

[4] T. Itoh and R. Mittra, "Spectral-domain approach for calculating dispersion characteristics of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 496–499, July 1973.

[5] P. Benedek and P. Silvester, "Microstrip discontinuity capacitances for right-angle bends, T-junctions, and crossings," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 341–346, May 1973.

[6] R. H. Jansen, "High-speed computation of single and coupled microstrip parameters including dispersion, high-order modes, loss, and finite strip thickness," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 75–82, Feb., 1978.

[7] R. F. Harrington, *Field Computation by Moment Methods*. New York: MacMillan, 1968, pp. 44.

Generalized Spectral Domain Method for Multiconductor Printed Lines and Its Application to Turnable Suspended Microstrips

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***Abstract*—An efficient method is developed for obtaining propagation characteristics of microstripline type structures in which a number of conductors are located on various interfaces. Specific computations have been carried out for suspended microstripline structures with tuning conductive septums. A number of data useful for design are included.**

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I. Introduction

THE SPECTRAL domain technique developed by Itoh and Mittra has been applied to a number of microstripline structures [1], [2]. It is an efficient numerical technique having several advantages over many other methods [3], [4]. However, to date, this technique has been applied only to the structures in which center conductors (strips) are located on one of the dielectric interfaces, e.g., the air-substrate interface.

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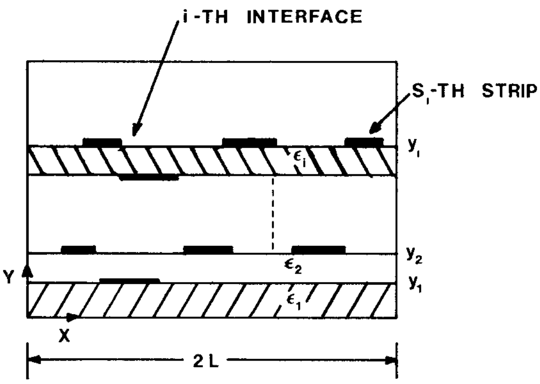

Fig. 1. Cross section of shielded multiconductor printed line.

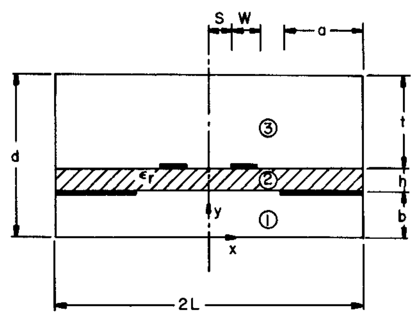

Fig. 2. Suspended microstripline with septums.

This paper reports a modification of the spectral domain technique which can handle the structures in which a number of conductors are placed on various interfaces (Fig. 1). The original version is not capable of solving such structures. Also, formulation in this paper is quite general and requires no structural symmetry to exist. Before discussing the technique, we will describe the motivation of the present work.

Recently, several attempts have been made to increase design flexibility of microwave integrated circuit structures by introducing additional conductors on interfaces different from the one on which the original strips are located. Aikawa reported the use of grounded septums located on the lower side of the substrate in the coupled suspended line [5] (Fig. 2). He has successfully developed tight couplers by adjusting the width of septums without which such couplers were extremely difficult to realize. Such composite structures are difficult to analyze, and the design procedure based on slowly convergent numerical methods is prohibitively expensive because there are more structural parameters to be adjusted than in conventional structures. Hence, development of an efficient analysis method is needed.

The principal purpose of the paper is to present a formulation for general structures (Fig. 1). Numerical results are presented for the suspended microstrip with two grounded septums. Most of the data are for the single strip case ($S=0$ in Fig. 2), as extensive data for coupled lines will be reported elsewhere.

II. Formulation

In this paper, we restrict ourselves to cases where the quasi-TEM approximation is valid, although the present method can readily be extended to a more rigorous dispersion analysis in which wave equations for inhomogeneous structures are treated. Under the quasi-TEM assumption, we only need to solve Laplace's equation in the cross section subject to appropriate boundary conditions. Instead of solving such a problem directly in the xy coordinate, we introduce the discrete Fourier transform of the potential

$$\tilde{\phi}(n,y)=\int_0^L \phi(x,y)\sin\left(\frac{n\pi}{2L}x\right)dx, \qquad n=1,2,\cdots,\infty \tag{1}$$

so that the partial differential equation (Poisson's equation) can be transformed to an ordinary differential equation

$$\left[\frac{d^2}{dy^2}-\left(\frac{n\pi}{2L}\right)^2\right]\tilde{\phi}(n,y)=0. \tag{2}$$

In (1) and (2) and throughout the rest of the paper n is the discrete Fourier transform variable. The solution of (2) in the ith layer is

$$\tilde{\phi}_i(n,y)=A_i(n)\sinh\frac{n\pi}{2L}y+B_i(n)\cosh\frac{n\pi}{2L}y \tag{3}$$

$\tilde{\phi}_i$ must be zero at both bottom and top of the shield case. In addition, when the conditions at the ith interface are Fourier transformed, we obtain

$$\tilde{\phi}_i(n,y_i)=\tilde{\phi}_{i+1}(n,y_i) \tag{4a}$$

$$\tilde{\phi}_i(n,y_i)=\tilde{\phi}_{Vi}+\tilde{\phi}_{0i} \tag{4b}$$

$$\epsilon_{i+1}\frac{d\tilde{\phi}_{i+1}}{dy}\bigg|_{y=y_i}-\epsilon_i\frac{d\tilde{\phi}_i}{dy}\bigg|_{y=y_i}=-\frac{\tilde{\rho}_i(n)}{\epsilon_0} \tag{4c}$$

where $\tilde{\rho}_i(n)$ is the transform of unknown charge distributions at the ith interface. $\tilde{\phi}_{Vi}$ is the transform of given potentials on the strips at the ith interface, whereas $\tilde{\phi}_{0i}$ is that of unknown potential distributions outside strips at the ith interface. Next, (3) is substituted into (4) for each i, and A_i's and B_i's are eliminated. After mathematical derivation for this process is completed, one obtains the following coupled algebraic equations:

$$\sum_{j=1}^{N}\tilde{G}_{ij}(n)\tilde{\rho}_j(n)=\tilde{\phi}_{Vi}+\tilde{\phi}_{0i}, \qquad i=1,2,\cdots,N \tag{5}$$

where $\tilde{G}_{ij}$'s are known. When there is no strip at the jth surface, the jth equation vanishes and the jth term on the left-hand side becomes zero as $\tilde{\rho}_j$ is zero for such j.

Notice that (5) is an $N\times N$ matrix equation in contrast to a set of $N\times N$ coupled integral equations which would appear in conventional space domain formulations that contain convolution integrals. $\tilde{G}_{ij}$ is actually the transform of the Green's function G_{ij} which determines the potential at the ith interface due to the unit charge at the jth interface. Also (5) contains a total of $2N$ unknown, $\tilde{\rho}_j$ and $\tilde{\phi}_{0i}$. However, N unknowns, $\tilde{\phi}_{0i}$, can be eliminated in the solution process and one can solve (5) only for N unknown $\tilde{\rho}_j$'s. To this end we apply Galerkin's method to (5). First we expand $\tilde{\rho}_j$ in the following manner:

$$\tilde{\rho}_j(n) = \sum_{s=1}^{S_j} \sum_{p=1}^{P_j} c_{jp}^s \tilde{\rho}_{jp}^s(n) \tag{6}$$

where S_j is the number of strips at the jth interface. $\tilde{\rho}_{jp}^s$ is the transform of an assumed charge distribution on the sth strip at the jth interface. Specific forms used for $\tilde{\rho}_{jp}^s$ will be described later.

Substituting (6) into (5) and taking the inner products of the resulting equations with $\tilde{\rho}_{iq}^v, q_N = 1, \cdots, P_i$ and $v = 1, \cdots, S_i$, one obtains the following $\sum_{j=1}^{N} P_j S_j \times \sum_{j=1}^{N} P_j S_j$ matrix equations for c_{jp}^s:

$$\sum_{j=1}^{N} \sum_{s=1}^{S_j} \sum_{p=1}^{P_j} K_{qp}^{vs}(i,j) c_{jp}^s = Y_q^v(i) \quad \begin{matrix} i = 1,2,\cdots,N \\ v = 1,\cdots,S_i \\ q = 1,\cdots,P_i \end{matrix} \tag{7}$$

where

$$K_{qp}^{vs}(i,j) = \sum_{n=1}^{\infty} \tilde{\rho}_{iq}^v(n) \tilde{G}_{ij}(n) \tilde{\rho}_{jp}^s \tag{8}$$

$$Y_q^v(i) = \sum_{n=1}^{\infty} \tilde{\rho}_{iq}^v(n) \left[\tilde{\phi}_{Vi}(n) + \tilde{\phi}_{0i}(n) \right]$$

$$= \sum_{n=1}^{\infty} \tilde{\rho}_{iq}^v(n) \tilde{\phi}_{Vi}(n). \tag{9}$$

The second infinite summation in (9) vanishes because by virtue of Parseval's relation

$$\sum_{n=1}^{\infty} \tilde{\rho}_{iq}^v(n) \tilde{\phi}_{0i}(n) = \frac{1}{2\pi} \int_0^L \rho_{iq}^v(x) \phi_{0i}(x) dx = 0. \tag{10}$$

In (10), we used the fact that the assumed charge distribution ρ_{iq}^v (the inverse transform of $\tilde{\rho}_{iq}^v$) is zero when x is outside the conductor whereas $\phi_{0i}(x)$ (the unknown potential) is zero on the conductor. Since $\tilde{\phi}_{Vi}(n)$ is the transform of given potential on the conductor, $Y_q^v(i)$ is known. It is also clear that $K_{qp}^{vs}(i,j)$ is also a known quantity. Hence K_{qp}^{vs} and Y_q^v can be computed quite efficiently once $\tilde{\rho}_{jp}^s$ is selected. Once c_{jp}^s's are obtained by solving (7), the charge distribution on the sth strip at the jth interface can readily be computed from

$$\rho_j(x) = \sum_{s=1}^{S_j} \sum_{p=1}^{P_j} c_{jp}^s \rho_{jp}^s(x) \tag{11}$$

where ρ_{jp}^s is the assumed charge distribution from which $\tilde{\rho}_{jp}^s$ was analytically derived.

Although (7) may seem complicated, in most cases it results in small size matrix, because for a reasonably accurate answer P_j only needs to be unity or at most two. For instance, when only one strip each is located at two different interfaces ($S_j = 1, N = 2$), the size of the matrix is either 2×2 or 4×4.

Before concluding this section, let us summarize the procedure for solution.

1) Select a set of functions, $\rho_{jp}^s(x)$, $p = 1,2,\cdots,P_j$, which individually reasonably represents the charge distribution on the sth strip at the jth interface. Note that they must be analytically Fourier transformable. For instance, in the computation for the suspended microstrip with septums, we have let $P_j = 1$ and chosen a Maxwell function for the charge on the strip and one half of a Maxwell function for the charge on the septum. These functions incorporate a correct singular behavior of the charge distributions at the edges.

2) Take the discrete Fourier transform of $\rho_{jp}^s(x)$ via (1) and obtain $\tilde{\rho}_{jp}^s(n)$ analytically. In the example, these functions are expressed in terms of Bessel functions.

3) Compute numerically $K_{qp}^{vs}(i,j)$ and $Y_q^v(i)$ via (8) and (9) as all quantities including $\tilde{\phi}_{Vi}$ are known analytically.

4) Solve (7) for c_{jp}^s and obtain $\rho_j(x)$ from (11).

III. Results for the Suspended Microstrips with Septums

Numerical results were obtained for a single suspended microstrip with septums ($S = 0$ in Fig. 2). In such structures, $\tilde{G}_{ij}$'s become

$$\tilde{G}_{11} = \frac{1}{\det} \left[\coth \hat{k}_n h + \frac{1}{\epsilon_r} \coth \hat{k}_n b \right] \tag{12a}$$

$$\tilde{G}_{12} = G_{21} = \frac{1}{\det} \frac{1}{\sinh \hat{k}_n h} \tag{12b}$$

$$\tilde{G}_{22} = \frac{1}{\det} \left[\coth \hat{k}_n h + \frac{1}{\epsilon_r} \coth \hat{k}_n t \right] \tag{12c}$$

$$\det = \epsilon_0 \hat{k}_n \Big[\epsilon_r + \coth \hat{k}_n h \coth \hat{k}_n b + \coth \hat{k}_n t \Big(\coth \hat{k}_n h + \frac{1}{\epsilon_r} \coth \hat{k}_n b \Big) \Big] \tag{12d}$$

where $\hat{k}_n = n\pi/2L$. As assumed charge distributions, we have chosen the ones having square integrable singularity at edges of strips.

First the accuracy of the method was checked by comparing our results with those reported by Aikawa [5] who used a finite difference technique. As shown in Fig. 3, the agreement is quite satisfactory.

A number of data are presented here for a single suspended line with symmetric septums. Figs. 4 and 5 present characteristic impedance and normalized guide wavelength, respectively, versus the width of the strip for a number of septum widths. It is seen from Fig. 5 that for large a, the guide wavelength λ_g becomes smaller as the strip width is increased. On the other hand, when a is reduced λ_g takes a maximum at some W. The reason for this phenomenon may be as follows. When a is large, the

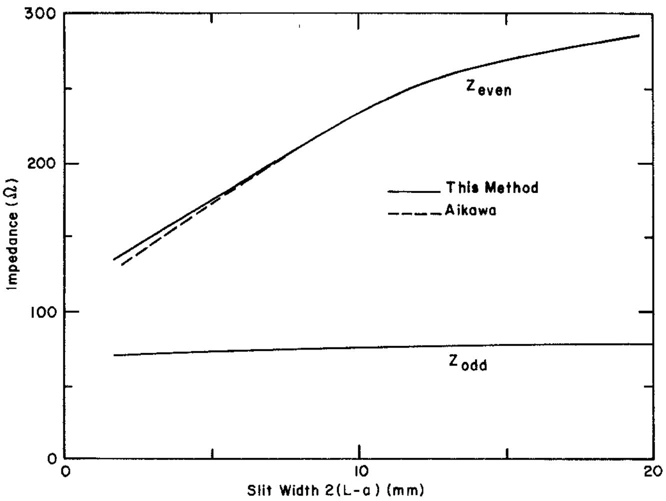

Fig. 3. Comparison of computed characteristic impedances with those by Aikawa. $\epsilon_r = 2.4$, $S = 0.335$ mm, $W = 1.48$ mm, $L = 16.4$ mm, $t = 16.4$ mm, $h = 1.64$ mm, $b = 8.2$ mm.

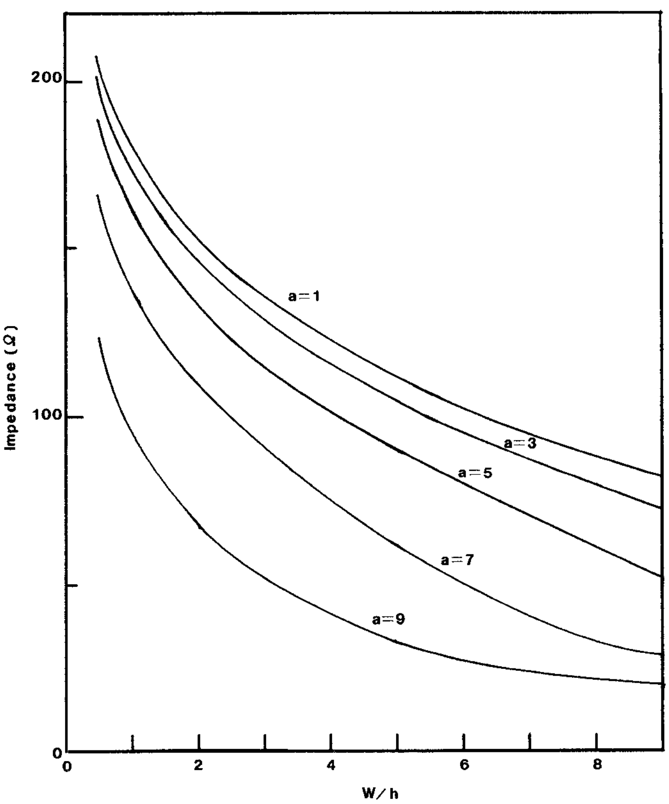

Fig. 4. Characteristic impedance versus the width of the strip. $\epsilon_r = 3.8$, $S/h = 0$, $L/h = b/h = t/h = 10$.

effect of the air portion (region 1) to the field distribution is reduced. As W is increased, most of the flux lies in the dielectric region, causing λ_g to be small. For small a, λ_g resembles that of the conventional suspended line. As W is increased, the effect of the air becomes more important and λ_g increases until the coupling between the strip and the septums becomes dominant. After such a situation a larger amount of flux moves into the dielectric region and λ_g becomes smaller again.

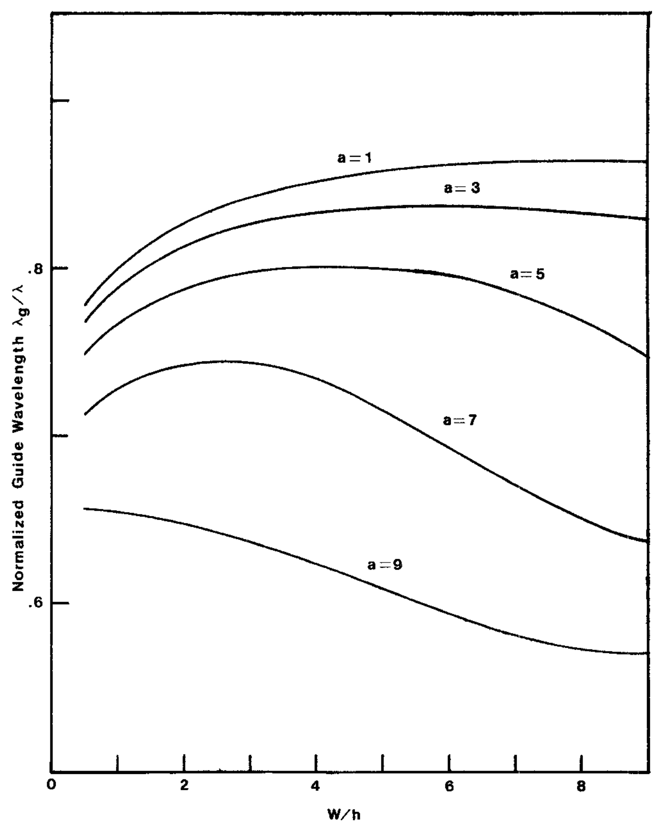

Fig. 5. Normalized guide wavelength versus the width of the strip. $\epsilon_r = 3.8$, $S/h = 0$, $L/h = b/h = t/h = 10$.

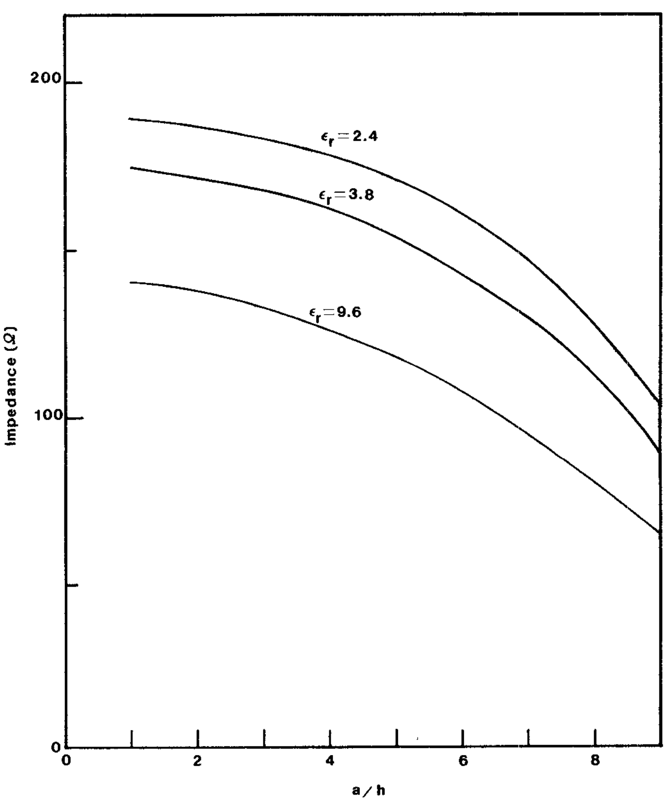

Fig. 6. Characteristic impedance versus the width of the septum. $S/h = 0$, $W/h = 1.2$, $L/h = b/h = t/h = 10$.

Fig. 6 shows characteristic impedance versus the septum width a for three different dielectric materials. The strip width W is fixed. It is clear that Z can be adjusted over a wide range by varying a. This feature is quite attractive in MIC application because in suspended line the fabrication of low impedance lines is often difficult [6].

IV. Conclusions

We presented a general method, based on spectral domain approach, for multiconductor printed lines for MIC. Numerical examples are given for the suspended microstrip with grounded septums. This structure is considered useful for MIC application, because propagation characteristics can be adjusted by septums which add one more degree of freedom in the design.

The numerical method presented here is applicable to a wide range of problems and has several advantageous features: 1) the method is numerically efficient, 2) no convolution integrals are involved, and 3) the size of the matrix is quite small.

References

[1] T. Itoh and R. Mittra, "A technique for computing dispersion characteristics of shielded microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 896–898, Oct. 1974.

[2] T. Itoh, "Analysis of microstrip resonators," *IEEE Microwave Theory Tech.*, vol. MTT-22, pp. 946–952, Nov. 1974.

[3] H. E. Stinehelfer, Sr., "An accurate calculation of uniform microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 439–444, July 1968.

[4] Y. L. Chow and I. N. El-Behery, "A dynamic spacial Green's function for microstrip lines," in *1978 IEEE Int. Microwave Symp. Digest*, pp. 341–343 (Ottawa, Canada), June 27–29, 1978.

[5] M. Aikawa, "Microstrip line directional couplers with tight coupling and high directivity," *Electronics and Communications in Japan*, vol. J60-B, no. 4, pp. 253–259, April, 1977.

[6] M. V. Schneider and W. W. Snell, Jr., "Harmonically pumped stripline down-converter," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 271–275, Mar. 1975.

Simulation Study of Electronically Scannable Antennas and Tunable Filters Integrated in a Quasi-Planar Dielectric Waveguide

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***Abstract*—Preliminary studies on electronically scannable leaky-wave antennas integrated in a dielectric waveguide are reported. Electronic scan is simulated by a small mechanical motion from which the relation between the scan angle and the necessary change in the dielectric constant can be derived. The work is also applicable to electronically tunable bandstop filters.**

I. Introduction

THIS PAPER presents an economical method useful as a preliminary study for the design of electronically scannable antennas and tunable filters in dielectric millimeter-wave integrated circuits. The results obtained by the present study can be helpful in establishing the requirement for the material in which the dielectric constant can be varied electronically. In addition, the method itself can be used to adjust the beam direction or scan the beam if speed is not of consideration.

A number of dielectric waveguides have been proposed for developing new types of millimeter-wave integrated circuits [1]–[3] which resemble optical integrated circuits. Recently, grating structures created in the dielectric waveguides have been used as leaky-wave antennas and bandstop filters [4]. The main-beam direction and the stopband are determined from the electrical length of the unit cell of gratings. In the leaky-wave antenna in [4], the beam was steered by changing the operating frequency. However, in actual application, often one would like to keep the frequency fixed and still need to steer the beam. Also, in the filter in [4], the stopband cannot be altered once the gratings are fabricated.

These problems may be overcome by incorporating an electronic phase shifter in the structure, which changes the electrical length for a fixed frequency. Recently, the use of a p-i-n layer incorporated in a submillimeter-wave dielectric waveguide was suggested as an electronic phase

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